# The Magnetization Vector

Recall that we defined the **Polarization vector** of a dielectric material as the **electric dipole density**, i.e.:

$$\mathbf{P}(\bar{r}) \doteq \lim_{\Delta v \to 0} \frac{\sum \mathbf{p}_n}{\Delta v} \qquad \left[\frac{\text{electric dipole moment}}{\text{unit volume}}\right]$$

Similarly, we can define a **Magnetization vector** $\mathbf{M}(\bar{r})$ of a material to be the density of **magnetic** dipole moments at location $\bar{r}$:

$$\mathbf{M}(\bar{r}) \doteq \lim_{\Delta v \to 0} \frac{\sum \mathbf{m}_n}{\Delta v} \qquad \left[\frac{\text{magnetic dipole moment}}{\text{unit volume}} = \frac{A}{m}\right]$$

Note if the dipole moments of atoms/molecules within a material are **completely random**, the Magnetization vector will be **zero** (i.e., $\mathbf{M}(\bar{r}) = 0$).

However, if the dipoles are **aligned**, the Magnetization vector will be **non-zero** (i.e., $\mathbf{M}(\bar{r}) \neq 0$)

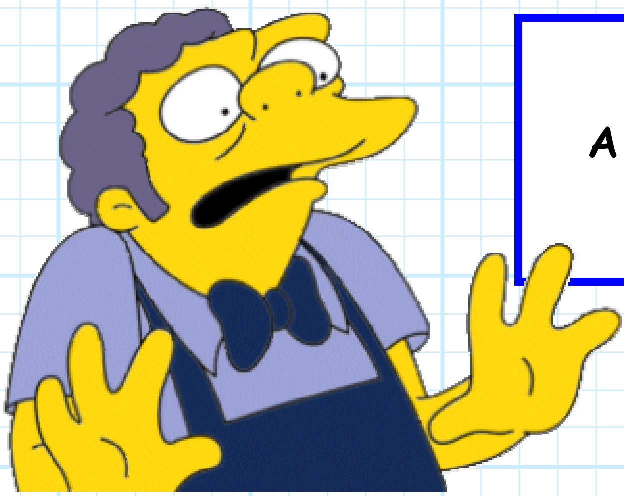
Recall a magnetic dipole will create a **magnetic vector potential** equal to:

$$\mathbf{A}(\bar{r}) = \frac{\mu_0 \mathbf{m} \times (\bar{r} - \bar{r}')}{4\pi |\bar{r} - \bar{r}'|^3}$$

Since the magnetic dipole moment of some **small** (i.e., differential) volume $dv$ of the material is:

$$\mathbf{m} = \mathbf{M}(\bar{r})\, dv$$

we find that the magnetic vector potential created by a **volume** $V$ of material with magnetization vector $\mathbf{M}(\bar{r})$ is:



$$\mathbf{A}(\bar{r}) = \iiint_V \frac{\mu_0 \mathbf{M}(\bar{r}') \times (\bar{r} - \bar{r}')}{4\pi |\bar{r} - \bar{r}'|^3}\, dv'$$

**Q:** *This is freaking me out!! I thought that **currents** $\mathbf{J}(\bar{r})$ were responsible for creating magnetic vector potential. In fact, I could have sworn that:*

$$\mathbf{A}(\bar{r}) = \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}(\bar{r}')}{|\bar{r} - \bar{r}'|}\, dv'$$

**A:** Relax, **both** expressions are correct!

Recall that we could attribute the electric field created by Polarization Vector $\mathbf{P}(\bar{r})$ to **polarization** (i.e., bound) **charges** $\rho_{vp}(\bar{r})$ and $\rho_{sp}(\bar{r})$, i.e., :

$$\rho_{vp}(\bar{\mathrm{r}}) = -\nabla \cdot \mathbf{P}(\bar{\mathrm{r}}) \qquad\qquad \rho_{sp}(\bar{\mathrm{r}}) = \mathbf{P}(\bar{\mathrm{r}}) \cdot \hat{a}_n$$

Similarly, we can **attribute** the magnetic vector potential (and therefore the magnetic flux density) created by Magnetization Vector $\mathbf{M}(\bar{r})$ to **Magnetization Currents** $\mathbf{J}_m(\bar{r})$ and $\mathbf{J}_{sm}(\bar{r})$.